The paradox $e = e^{1+2\pi i}$ (1) thus $e = (e^{1+2\pi i})^{(1+2\pi i)} = e^{(1+2\pi i)(1+2\pi i)} = e^{1+4\pi i-4\pi^2} = e^{1-4\pi^2}$

The first thing to note is that when we write w^z we are really referring to a multivalued function. This can be seen by expressing it as $e^{wLogz} = e^{w(lnz+n2\pi i)}$ where I use Log to denote the multivalued logarithm and ln for the principal (n=0) value. So really 'w^z' can thought of as a function W[z] mapping z onto a myriad of values.

Applying this formalism to the case $E(z) = e^{zLog(e)} = e^{z[ln(e)+k2\pi i]}$ where I write $E(z)$ instead of e^z to emphasise I'm thinking of it here as the multifunction and not the complex number. E(z) has a spectrum of values for each z. Consider $E(1+2\pi i) = e^{(1+2\pi i)[ln(e)+k2\pi i]}$. If we now choose the PV (Log(e)=ln(e)=1, corresponding to $k = 0$), we find that $E(1+2\pi i) = e^{(1+2\pi i)}$, the RHS is now the value of the multifunction and is the complex number $e^{1+2\pi i} = e$ by Euler identity.

Thus in order to equate $E(1+2\pi i) = e$ we had to assume the branch corresponding to $Log(e) = 1$

Take logarithms of both sides of (1) also: $Log(e) = Log(e^{1+2\pi i})$ we've assumed that $Log(e) = 1$ in order for this relation to hold so it must be true that $1 = Log(e^{1+2\pi i})$ (1b)

Next consider $(w^a)^b = w^{ab}$ (2). The RHS can be written as $e^{abLog(w)}$ and once we fix Log(w)=k then if we want to have the equality in (2) hold, then by previous exercise in Penrose we are forced into a fixing in the LHS, i.e., the LHS can be written as $e^{bLog(w^a)}$ and for equality to hold we must have $Log(w^a) = ak$, only then is (2) valid.

But now examine this for our actual case: $(e^{1+2\pi i})^{(1+2\pi i)} = e^{(1+2\pi i)(1+2\pi i)}$. The RHS can be written as $e^{(1+2\pi i)(1+2\pi i)Log(e)}$, whilst the LHS can be written $e^{(1+2\pi i)Log[e^{1+2\pi i}]}$. If we assume on the RHS we take Log(e)=q, then on the LHS we are forced(if we want equality in (2)) to take $Log[e^{1+2\pi i}] = (1+2\pi i)q$. q must actually be ln(e)=1, and so $Log[e^{1+2\pi i}] = (1+2\pi i)$

However by (1b) we found that $Log[e^{1+2\pi i}] = 1$, so it appears we can't satisfy both (1) and (2) at once.