

Exercise [16.19]

The set $(A^B)^C$ comprises all mappings F from C to the set of mappings $f: B \rightarrow A$, i.e.

$$(A^B)^C = \{F_c \mid c \in C, F_c \in \{f: B \rightarrow A \mid \forall b \in B \ f(b) = a \in A\}\}$$

The set $A^{B \times C}$ comprises all mappings ψ from $(B \times C)$ to A , i.e.

$$A^{B \times C} = \{\psi: (B \times C) \rightarrow A \mid \forall b \in B, c \in C \ \psi(b, c) = a \in A\}$$

For any $c \in C$ you can then identify $F_c \in (A^B)^C$ with $\psi(\cdot, c) \in A^{B \times C}$.