

Exercise [16.14]

I don't know if this is the solution that is looked for, but there is a contradiction between

- the assumption of a "set of all sets"
- and Cantors's proof of $\alpha < 2^\alpha$:

Assume that the **set A of all sets** exists. Let S be a subset of A . As A contains all sets, there must be an $x \in A$ with $x = S$. Hence there is a 1–1 mapping

$$f: S \mapsto x \quad \forall S \in 2^A$$

of 2^A onto a subset of A (a mapping from the set of all subsets of A onto the subset of A that contains all subsets of A). With α being the cardinality of A , this would mean that $2^\alpha < \alpha$ in contradiction to Cantor's proof.