In exercise [12.12], the conclusion $d(d\Phi) = 0\ for\ \Phi$ of degree 0 can be proved directly:

$$d(d\Phi) = d(\sum_i (\partial\Phi/\partial x_i) dx_i) = \sum_i \sum_j (\partial^2\Phi/dx_i dx_j) dx_i \wedge dx_j = 0$$

because $dx_i \wedge dx_j$ is antisymmetric but the second partial derivative $(\partial^2\Phi/dx_i dx_j)$ is symmetric.

Establishing this result directly for degree zero permits one to establish it for all degrees by induction using the second of the two defining relations given immediately above in RtR.