I had a thought that complements the posted solution to Exercise 12.10 – namely what thinking of the dxdy as dx∧dy gets you. (This may be part of what Penrose had in mind for this exercise.)

Let $x = r\cos\varphi, y = r\sin\varphi$, so

$$\begin{aligned} dx \wedge dy &= (\cos\varphi dr - r\sin\varphi d\varphi) \wedge (\sin\varphi dr + r\cos\varphi d\varphi) \\ &= r(\cos\varphi)^2 dr \wedge d\varphi - r(\sin\varphi)^2 d\varphi \wedge dr \\ &= r dr \wedge d\varphi \end{aligned}$$

In short, the antisymmetry generates the Jacobean, which is a determinant, ie, an antisymnmetric sum of products