Given any function $\alpha(x, y)$ which satisfies $\nabla^2\alpha = 0$, let's construct a function β as follows:

$$\beta = \int \frac{\partial\alpha}{\partial x} dy \qquad (1)$$

So, differentiating (1) with respect to y gives:

$$\frac{\partial\beta}{\partial y} = \frac{\partial\alpha}{\partial x} \qquad (2)$$

Differentiating (1) with respect to x gives:

$\dfrac{\partial\beta}{\partial x} = \dfrac{\partial}{\partial x} \displaystyle\int \dfrac{\partial\alpha}{\partial x} dy$, or

$$\frac{\partial\beta}{\partial x} = \int \frac{\partial^2\alpha}{\partial x^2} dy \qquad (3)$$

Now since $\nabla^2\alpha = \dfrac{\partial^2\alpha}{\partial x^2} + \dfrac{\partial^2\alpha}{\partial y^2} = 0$:

$$\frac{\partial^2\alpha}{\partial x^2} = -\frac{\partial^2\alpha}{\partial y^2}$$

and so (3) can be written as:

$$\frac{\partial\beta}{\partial x} = \int \frac{\partial^2\alpha}{\partial x^2} dy = -\int \frac{\partial^2\alpha}{\partial y^2} dy = -\frac{\partial\alpha}{\partial y} \qquad (4)$$

And so (2) and (4) are the Cauchy-Riemann equations for a function

$\Phi = \alpha + i\beta$, where the functions α and β are as defined above.