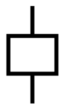
Exercise [13.26]

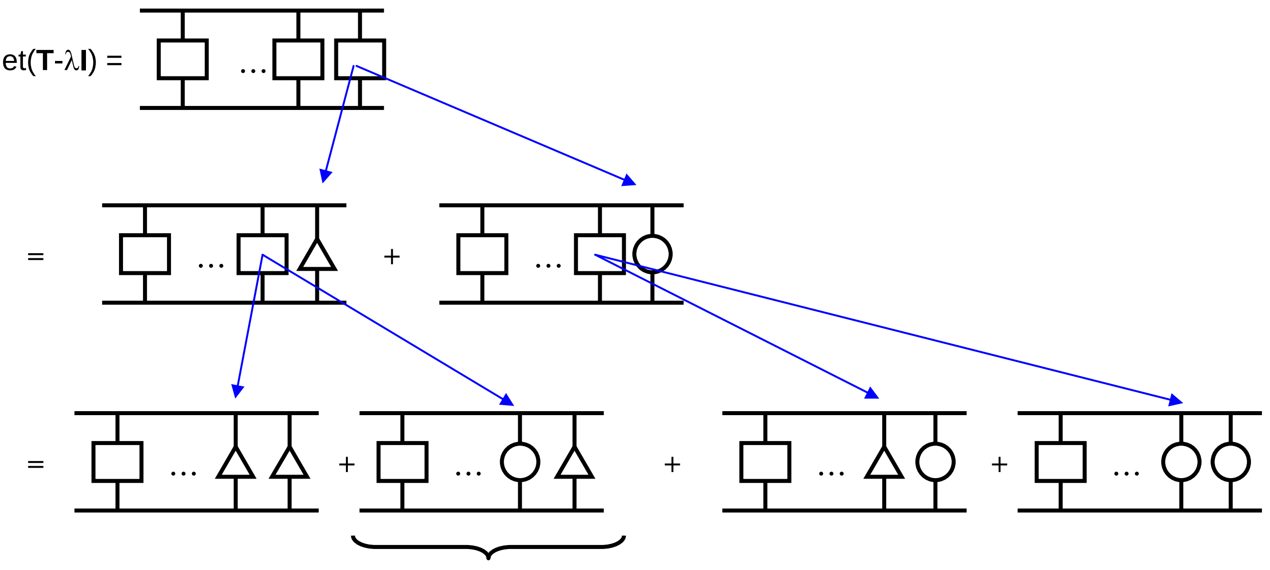
Here is my proposal for the diagrammatic calculation of the polynomial det(**T**-λ**I**):

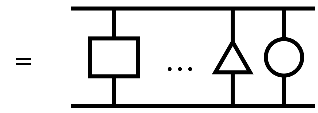
Convention:

 $= T^a{}_b$  $= (-\lambda)\cdot$ $= (-\lambda)\cdot\delta^a{}_b$  $= T^a{}_b + (-\lambda)\cdot\delta^a{}_b$

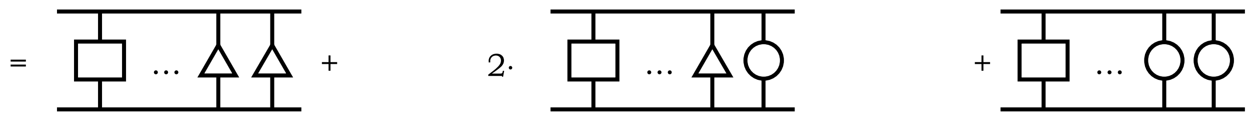
We then have

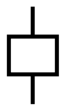
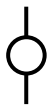
n!·det(**T**-λ**I**) =

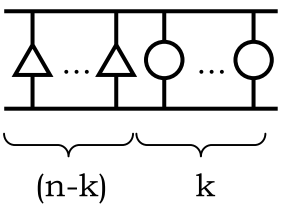
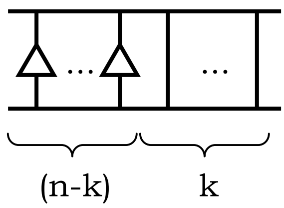




(swapping top and bottom index lines of triangle and circle produces one (-1)sign each, which cancel; hence these elements commute)



The above drawings show that successively resolving all n  into  and  works the same way as successively multiplying out all brackets in the binomial formula $(x+y)^n$. We can therefore conclude that

det(**T**-λ**I**) = $\frac{1}{n!}\sum_{k=0}^{n}\binom{n}{k}$  $= \frac{1}{n!}\sum_{k=0}^{n}\binom{n}{k}(-\lambda)^k$ 

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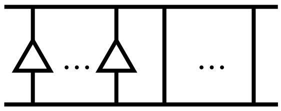
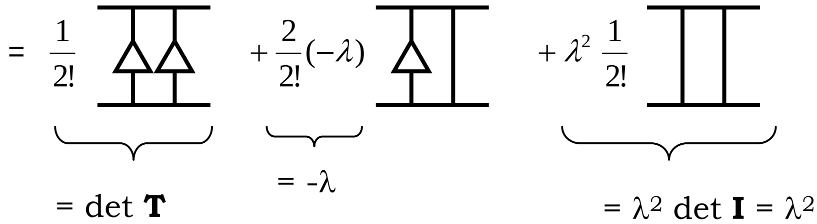
Applying this result to the cases n=2 and n=3 yields:

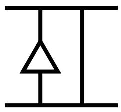
<u>Case n=2</u>:

$$\det(\mathbf{T}\text{-}\lambda\mathbf{I}) = \frac{1}{n!}\sum_{k=0}^{2}\binom{2}{k}(-\lambda)^k$$





= det **T** - λ

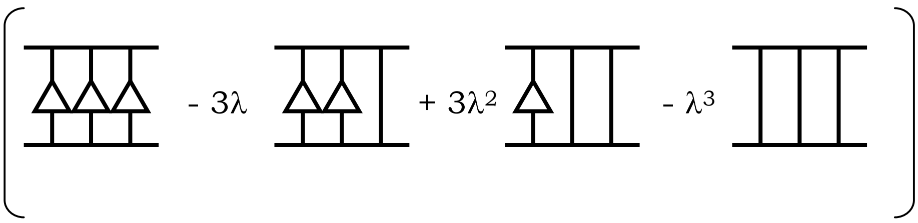


+ λ^2

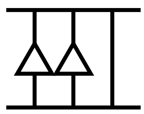
(I don't know if there is a way to simplify this further, based on diagrammatic rules.)

<u>Case n=3</u>:

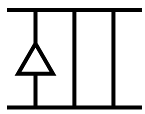
$$\det(\mathbf{T}\text{-}\lambda\mathbf{I}) = \frac{1}{3!}$$



= det **T** $-\frac{\lambda}{2}$



$+\frac{\lambda^2}{2}$



$- \lambda^3$