I was a bit puzzled by Exercise 20.8, where you're talking about the Taylor expansion of the Hamiltonian H to second order at equilibrium, and you say that there aren't second order terms in $p_i q_j$.

"In theory" you could have $H = p^2 - pq + q^2$, which has a minimum at $p = q = 0$ and is positive for all other (p, q), where p is a generalized momentum and q is a generalized coordinate.

I thought the Hamiltonian for a particle in an electromagnetic field might be a counterexample. It really wasn't, but it gave me a clue: if you get the Hamiltonian from the Lagrangian, terms in the Lagrangian that look like $\dot{q}_j f(q_i)$ are eliminated. And at equilibrium, it's not p_i that is 0 in general, it's $\dot{q}_i$.

It was not clear to me, reading the book. You say you're talking about a general Hamiltonian system (pg 480 in the 2004 hardcover ed., just before exercise 20.8). But do we know that the Hamiltonian is neatly divided into a potential energy that's a function only of position, and a kinetic energy, that's a function only of velocities? You don't say it is, and the potential could be velocity-dependent in general. Are you really talking about a mechanical system, without electromagnetic fields, say?

Also, I think it would be a good idea to mention that the Hamiltonian evolution equations $\Rightarrow$ energy conservation when you introduce them. It's an important feature of the Hamiltonian, that it's about a closed system, you wouldn't have viscous forces decreasing the energy. It would help to make sense of the evolution equations.

If you said there are no terms in the Taylor expansion that look like $\dot{q}_i q_j$, and that the equilibrium is at $\dot{q}_i = 0$, not $p_i = 0$ in general, it would be generally true (I think) and be a much easier exercise.

By the way I figured out what makes the magic circle "tick" :) Laura