$\Phi(z) = \alpha + i\beta$, where $\alpha = \alpha(x, y)$ and $\beta = \beta(x, y)$.

From the definition of the derivative, we can write

$\dfrac{d\Phi}{dz} = \lim\limits_{\Delta z \to 0} \dfrac{\Phi(z + \Delta z) - \Phi(z)}{\Delta z}$, where $\Delta z = \Delta x + i\Delta y$.

Now $\lim\limits_{\Delta z \to 0} \dfrac{\Phi(z + \Delta z) - \Phi(z)}{\Delta z} =$

$$\lim_{\Delta z \to 0} \frac{\alpha(x + \Delta x, y + \Delta y) - \alpha(x, y) + i\{\beta(x + \Delta x, y + \Delta y) - \beta(x, y)\}}{\Delta x + i\Delta y} \qquad (1)$$

Remember from the definition of derivative that $\lim\limits_{\Delta z \to 0}$ must be the same *regardless of the manner in which* $\Delta z \to 0$.

So let $\Delta z \to 0$ along the x direction, so that $\Delta y = 0$ and $\Delta z = \Delta x$.
We can then write (1) as

$\lim\limits_{\Delta z \to 0} \dfrac{\alpha(x + \Delta x, y) - \alpha(x, y) + i\{\beta(x + \Delta x, y) - \beta(x, y)\}}{\Delta x}$ or as

$$\lim_{\Delta z \to 0} \frac{\alpha(x + \Delta x, y) - \alpha(x, y)}{\Delta x} + i \lim_{\Delta z \to 0} \frac{\beta(x + \Delta x, y) - \beta(x, y)}{\Delta x}$$

$$= \frac{\partial \alpha}{\partial x} + i\frac{\partial \beta}{\partial x} \qquad (2)$$

In a similar manner let $\Delta z \to 0$ along the y direction, so that $\Delta x = 0$ and $\Delta z = \Delta y$. We can then write (1) as

$\lim\limits_{\Delta z \to 0} \dfrac{\alpha(x, y + \Delta y) - \alpha(x, y) + i\{\beta(x, y + \Delta y) - \beta(x, y)\}}{i\Delta y}$ or as

$$\lim_{\Delta z \to 0} \frac{\alpha(x, y + \Delta y) - \alpha(x, y)}{i\Delta y} + i \lim_{\Delta z \to 0} \frac{\beta(x, y + \Delta y) - \beta(x, y)}{i\Delta y}$$

$$= -i\frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial y} \qquad (3)$$

Since (2) and (3) must be equal, it follows that

$\dfrac{\partial \alpha}{\partial x} = \dfrac{\partial \beta}{\partial y}$ and $\dfrac{\partial \beta}{\partial x} = -\dfrac{\partial \alpha}{\partial y}$

which are the Cauchy-Riemann equations.